**Ch 12 - Methods of Proof for Quantifiers**

* **Goal:** present methods of proof that allow us to prove all and only the first-order validities, and all and only the first-order consequences of a given set of premises.
  + ie, to devise methods of proof sufficient to prove everything that follows in virtue of the meanings of quantifiers, identity, and the truth-functional connectives

**Valid Quantifier Steps**

**Universal Elimination (aka Universal Instantiation)**

* Suppose we have established ∀x S(x), and we know that c names an object in the domain of discourse.
* We may infer S(c).
* There is no way the universal claim could be true without the specific claim also being true
* We move from a step with a quantifier to one without the quantifier.

**Existential Generalization (aka Existential Introduction)**

* If we have established a claim of the form S(c) then we may infer ∃x S(x)

**Existential Elimination (aka Existential Instantiation)**

* If we have proven ∃x S(x), then we can give a name, say c, to one of the objects satisfying S(x), and we may assume S(c) and use this in our proof
* **example**
  + We have shown that there is a prime number between n and m. Call it p.
* **example**
  + Let p be such a prime number.

**Method of General Conditional Proof**

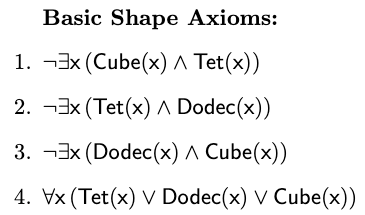
* involves reasoning about an arbitrary object of a particular kind in order to prove a universal claim about all such objects
  + similar to conditional proof
  + similar in spirit to method of existential instantiation
* method of reasoning used much in math
  + suppose we want to prove ∀x (P(x) *→* Q(x)) from some premises
  + choose a name not in use, say c, assume P(c) and prove Q(c)
  + infer the desired result
* **example:** Prove that the square root of every prime number is irrational
  + to apply general conditional proof, assume that p is an arbitrary prime number
  + show that sqrt(p) is irrational
  + infer the general claim
  + **proof**
    - let p be a prime number, ie any prime number
    - therefore if p divides a square, say k^2, then it divides k
    - if p divides k^2, then p^2 also divides k^2
    - assume that sqrt(p) is rational
      * write it in lowest terms n/m
      * square both sides
      * p = n^2/m^2
      * hence pm^2=n^2
      * therefore p divides n^2, so p divides n and p^2 divides n^2
      * therefore p^2 divides pm^2, so p divides m^2
      * therefore p divides m
      * so p divides both n and m contradicting the statement that n/m is p written in lowest terms
    - by contradiction, sqrt(p) is irrational

**Universal Generalization**

* in formal systems of deduction, the method of general conditional proof is usually broken down into two parts
  + conditional proof
  + method for proving completely general claims, of the form ∀x S(x), called **universal generalization** or **universal introduction**
    - if we are able to introduce a new name c to stand for a completely arbitrary member of the domain of discourse and go on to prove the sentence S(c), then we can conclude ∀x S(x)
* any proof using general conditional proof can be converted into a proof using universal generalization

**Axiomatizing Shape**

* We started a project of **giving axioms for the shape properties in Tarski’s World.**
* So far, we have axioms that describe basic facts about the three shapes
* **How to choose which sentences to take as axioms?**
  + **Main criterion:** correctness
    - the axioms must be true in all relevant circumstances, either in virtue of the meanings of the predicates involved, or because we have restricted our attention to a specific type of circumstance
  + **Second criterion:** completeness
    - a set of axioms is complete if, whenever n argument is intuitively valid (given the meanings of the predicates and the intended range of circumstances), its conclusion is a FO consequence of its premises taken together with the axioms in question
  + **These two notions are not precise.** They depend on vague notions of meaning and “intended circumstances”
* **Recall the basic shape axioms**

****

* the first three are correct in virtue of the meanings of the predicates
* the fourth expresses a truth about all worlds of the sort that can be built in Tarski’s world
* The set of four axioms is complete if we restrict attention to the three shape predicates.
* If we use the predicate **SameShape** then the set of four basic axioms is not complete
  + this means that given an argument that is intuitively valid cannot be turned into one that is FO valid just with the four axioms
  + we need more axioms that describe the predicate SameShape
  + in particular we can use intro and elim inference rules for this predicate